We introduce a new model of competition on growing networks. In particular, we couple type adoption with preferential attachment. Main qualitative feature of the model: often competitors will coexist.

Graph evolution:
- linear preferential attachment
  - $G_0 = (V_0, E_0)$ initial graph
  - $m$ new edges w/each node
  - $P$ (connect to $v$) = $\frac{\deg(v)}{\sum_x \deg(x)}$

Type adoption: via initial connections and randomness
- Initial nodes have a type/color out of $N$ possible ones.
- Incoming nodes get a type when they enter the network:
  - $u = $ # initial neighbors of type $i$
  - $P$ (new node is of type $i$ | $u$) = $p_{ui}$
  - $\{p_{ui}\}_{i=1}^N$ are parameters of the model.
- When $N = 2$ (red/blue), let $p_k := P(\text{red} | k \text{ red neighbors})$.

Intuition
- Three or more types. In the linear model the same results apply. In nonlinear models, the evolution of the fractions of types is governed by an ODE driven by a vector field $P$, which is the multidimensional analogue of the polynomial $P$. The behavior of this ODE in general is open.
- Changing preferences.
- Allowing multiple types for a single individual.
- Incorporating marketing / strategic considerations.

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Motivation/application
- Sprint
- Mobile
- AT&T
- T-Mobile
- Polyvore

Theorem (Linear model).
Suppose that $p_k = k/m$ for all $0 \leq k \leq m$, and that initially there are nodes of both colors. Then the fraction of red nodes, $x_n$, converges almost surely to a random point in $[0,1]$.

Further directions
- Three or more types. In the linear model the same results apply. In nonlinear models, the evolution of the fractions of types is governed by an ODE driven by a vector field $P$, which is the multidimensional analogue of the polynomial $P$. The behavior of this ODE in general is open.
- Changing preferences.
- Allowing multiple types for a single individual.
- Incorporating marketing / strategic considerations.

Main results
We are interested in the fraction of nodes of each type. In the case of two colors (red/blue) we provide a complete phase diagram of the asymptotic behavior of the process.

Phase diagrams when there is no bias towards either color ($p_k = k/m$) and initially there are nodes of both colors. Then the fraction of red nodes, $x_n$, converges almost surely to a random point in $[0,1]$. Furthermore, the limiting distribution has full support on $[0,1]$ and no point masses.

Theorem (Nonlinear models).
Suppose that $p_k \neq k/m$ for some $0 \leq k \leq m$, and initially there are nodes of both colors. Then the fraction of red nodes converges a.s. to a random point in the finite zero set of the polynomial

$$P(z) = \frac{1}{2} \sum_{k=0}^m \binom{m}{k} z^k (1-z)^{m-k} \left( p_k - \frac{k}{m} \right)$$